

Multi-Modal Medical Image Denoising using Wavelets: A Comparative Study

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In medical image processing Noise removal is an important step for recreating a high-quality image like X-ray, ultrasound, MRI etc. While acquiring, transmitting, and retrieving from storage devices normally images are degraded due to noises like Gaussian, Speckle etc. So, noise must be removed from the images for proper diagnosis. Researchers are still looking for an effective noise reduction means. Wavelet Transform (WT) is considered as a powerful transform method for removal of noise. For denoising of medical images affected by Gaussian noise, various wavelets have been proposed. In this paper, various wavelets are used to study the denoising multi-modal medical images affected by Gaussian noise. Here, proposed wavelet gives better results than the wavelets which have been implemented so far now. Denoising results of medical images are compared on the basis of Root Mean Square Error (RMSE), Signal-Noise Ratio (SNR), Peak Signal-Noise Ratio (PSNR) and execution time (TE).

Keywords: CT – Scan, Medical Image Denoising, MRI, Wavelet Transform (WT), Ultrasound, X-ray.

Medical imaging is an imaging of various body parts. It includes radiology, thermograph, endoscope, medical photography, and microscopy.¹ It plays a main part for diagnosis, research related findings etc. These images are generally get noisy while acquisition and transmission of the images. Various factors like various noises, disruption due to blood movement, blood flow, body fat and breathing motion etc. are responsible for distortion and corruption of medical images. So it is very essential to denoise the image for enhancing the image class. Researchers are still looking for an effective image denoising means.² To remove noise from image is a repairing process. By using prior

information of the degradation process, efforts are made to recover an image that gets corrupted.³ Various noise like Gaussian noise, Impulse noise, periodic noise etc. affect medical images. There are mainly four denoising methods which are available in literature. These denoising approaches are (i) filtering method, (ii) transform domain method (iii) statistical method and (iv) Machine Learning (ML) Methods. Filtering approach denoise the images by using filter directly on corrupted image. In transform-domain filtering like Fourier Transform and Wavelet Transform convert the spatial realm data to the frequency realm. And filtering operations are executed in frequency

realm. In Statistical Approach, images are generally modelled with the help of a Rician distribution. Here, noise variance is an essential factor in noise removal. In machine learning approach, computer algorithms which have the property of self-learning and enhance itself repeatedly through training and by the use of data are used for denoising.⁴ Every approach or method for denoising has its own assumptions, merits and demerits.

Discrete Wavelet Transform (DWT): It is a transform domain method. Other transform techniques are Fourier Transform (FT), Wavelet transform (WT), Curvelet transform (CT) etc. The Discrete Wavelet Transform (DWT) of image gives a non-redundant data. It gives enhanced spatial and spectral localization of image construction. Here, original image is given to two complementary filters which provides two signals, approximation and details. It is disintegration or investigation of image. The data can be reconstructed back into the original image without losing details. This is reconstruction or synthesis process of image. This mathematical process of analysis and synthesis, is discrete wavelet transform and inverse discrete wavelet transform. Using DWT, an image can be decomposed into a series of images having different spatial resolution. In 2D image, an N level decomposition is executed which gives 3N+1 different frequency bands. Here, the Gaussian noise will almost average out in low frequency wavelet coefficients. While in the high frequency levels, wavelet coefficients can be threshold.⁵

Performance Evaluation in Image Denoising

Subjective method and objective method are two types of techniques for image quality assessment (IQA). In subjective method, image quality is decided by the human beings. And to measure the image quality, it is considered as the most correct and reliable method. But, this process is very sluggish, difficult and expensive for implementation. So, the objective image quality metrics which automatically calculate the image quality is quite convenient. The purpose of IQA is to calculate the image quality which should be very close to the subjective assessment. So, the second method is preferred. It includes mathematically defined measuring parameters like Mean Square Error (MSE), Root Mean Square Error (RMSE) and Peak Signal-to-Noise Ratio (PSNR).⁴ Under objective method, Root Mean Square Error (RMSE) is

frequently used. For image quality measurement, it is considered as a highly reliable. Mathematically: let the actual image, noisy image and the denoised image be denoted by $i(x,y)$, $n(x,y)$ and $i'(x,y)$ respectively. And, the discrete spatial coordinates of the images are represented by x and y . Assume the size of image be $M \times N$ pixels i.e., $x = 1, 2, \dots, M$ and $y = 1, 2, \dots, N$. Then, the MSE and RMSE can be defined as

$$MSE = \sum_{x=1}^M \sum_{y=1}^N \frac{[i'(x,y) - i(x,y)]^2}{MN} \dots(1)$$

$$RMSE = \sqrt{MSE} \dots(2)$$

Second image quality measurement parameter is Peak Signal to Noise ratio (PSNR). PSNR is inversely proportional to RMSE and its unit is in db (decibels). It is defined as the ratio of Peak Signal Power to Noise Power. It is mathematically defined by

$$PSNR = 20 \log_{10} [255 / RMSE] \dots(3)$$

here 255 is the Maximum Pixel Value for an 8 bits/gray-scale image.

It compares the quality of reconstructed image and the original image. It gives a single number which indicates the class of new image. Denoised images having lesser MSE and greater PSNR are considered superior.

LL ³	LH ³	LH ²	LH ¹
LH ³	HH ³		
HL ²		HH ²	HH ¹
HL ¹			

1, 2, 3 ----- Decomposition levels
 H ----- High frequency bands
 L ----- Low frequency bands

Fig. 1. 2D-DWT with 3-Level decomposition

Execution time (ET) is defined as the time taken by a digital computing platform to execute the filtering algorithm when no other software, except the operating system (OS), runs on it. Lesser is the time, better is the measuring parameter.^{4,6,7,8}

Literature Review

Various wavelets for image denoising have been proposed by researchers. Nadir Mustafa *et al.*⁹ proposed bi-orthogonal wavelet which is

found to be more effective method than other wavelet families such as Haar, Daubechies, and Symlets. It gave better mean square error (MSE) in soft and hard threshold. Sugandha Agarwal *et al.*² based on the statistical measures and visual quality of MRI image proposed Symlet based Wavelet Transform which outperformed other wavelet transforms. In this paper, the efficiency of various wavelet family i.e. Haar, Morlet, Symlet,

original image for WT

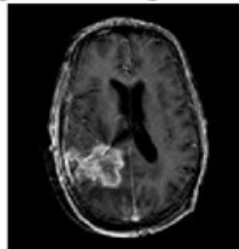


Fig. 2(a) Brain MRI image

Noise variance = 0.05



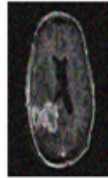
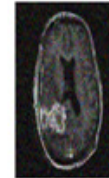
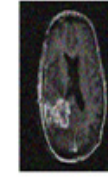
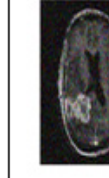
Noisy Image	Wavelets				
	1. Harr	2. db4	3. sym4	4. bior6.8	5. dmey
noisy image 	Denoised Img 	Denoised Img 	Denoised Img 	Denoised Img 	Denoised Img 

Fig. 2(b)

Noise variance = 0.09




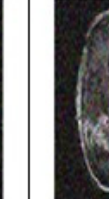


Noisy Image	Wavelets				
	1. Harr	2. db4	3. sym4	4. bior6.8	5. dmey
noisy image 	Denoised Img 	Denoised Img 	Denoised Img 	Denoised Img 	Denoised Img 

Fig. 2(c)

Fig. 2 (a-c). MRI Denoised Images by various wavelets

Daubechies were used for denoising the speckle noise from MRI of brain image. According to R. Sujitha *et al.* [10] the haar wavelet (db1) gave the best results as compare to other wavelets for Simulated & MRI image. S.Kother Mohideen *et al.*⁵ mentioned wavelet coiflet for better image denoising. As per Ajeet Singh¹¹, the best PSNR is obtained at the decomposition level of two.

Wavelets

Wavelets are generally categorized into three parts: continuous, discrete, and multiresolution-based. In continuous wavelet transform (CWT), a given finite energy signal is

projected on a continuous family of frequency bands. Discrete Wavelet Transform (DWT) is one more form of representation of the signal. It does not vary the data. It is a sampled form of the CWT. The reconstructed data contains high redundancy. DWT is more effective in removing redundancy than CWT.¹³ The DWT decomposes the original signal into an approximation subsignal and detailed subsignals. While the Multiresolution Analysis (MRA) algorithm continue to decompose the approximation subsignal, which again gives detailed subsignals and an approximation subsignal. The choice of the decomposition level

original image



Fig. 3 (a) Chest X-ray

Noise variance = 0.05

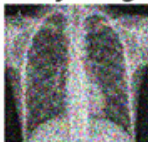





Noisy Image	Wavelets				
	1. Harr	2. db4	3. sym4	4. bior6.8	5. dmey
noisy image 	Denoised Im 	Denoised Im 	Denoised Img 	Denoised Img 	Denoised Imc 

Fig. 3(c)

Noise variance = 0.09

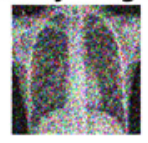





Noisy Image	Wavelets				
	1. Harr	2. db4	3. sym4	4. bior6.8	5. dmey
noisy image 	Denoised Im 	Denoised In 	Denoised Img 	Denoised Img 	Denoised Imc 

Fig. 3(c)

Fig. 3(b-c). X-ray Denoised Images by various wavelets

is based on the earlier size of the original data, and the requisite spectral and temporal resolution. Lastly, the original data can be reconstructed by the approximation coefficients of the last decomposition level and the accumulated detailed coefficients of all decomposition levels.¹⁴ In this paper, wavelets like Harr, db4, sym4, bior6.8 and dmey which were proposed and studied by the researchers are used for denoising.

a) Harr wavelet: Harr wavelet is a square-shaped function which can be rescaled. Its mother wavelet function $\psi(t)x(t)$ can be described as:

$$\begin{aligned} x(t) &= 1, 0 \leq t < 0.5 \\ &= -1, 0.5 \leq t < 1 \\ &= 0, \text{ otherwise} \end{aligned}$$

b) Daubechies (db4) wavelet: Ingrid Daubechies did the basic research work for this wavelet. These are orthogonal wavelet group. DWT is defined by db4 wavelets. Each wavelet is having scaling factor (named as the father wavelet) which gives an orthogonal multiresolution analysis.

c) Symlet (sym4) wavelet: Daubechies' least-asymmetric wavelets are also known as symN wavelets. These wavelets are more symmetric. In symN, N is the number of vanishing moments.

d) Biorthogonal (bior6.8) wavelet: These wavelets have two functions: one is scaling functions and another is related to scaling filters. One function is for analysis while another function is for synthesis. They can have dissimilar numbers of vanishing moments and regularity characteristics.



Fig. 4 (a) Original CT scan of Chest

Noise variance = 0.05

Noisy Image	Wavelets				
	1. Harr	2. db4	3. sym4	4. bior6.8	5. dmey
noisy image 	Denoised Img 	Denoised Img 	Denoised Img 	Denoised Img 	Denoised Img

Fig. 4 (c)

Noise variance = 0.09

Noisy Image	Wavelets				
	1. Harr	2. db4	3. sym4	4. bior6.8	5. dmey
noisy image 	Denoised Img 	Denoised Img 	Denoised Img 	Denoised Img 	Denoised Img

Fig. 4 (d)

Fig. 4(b-c). CT Denoised Images by various wavelets

e) Meyer (dmey) wavelet: The Meyer wavelet is a frequency-band-limited orthogonal wavelet. In 1985, it was suggested by Yves Meyer. They are indeterminately differentiable orthonormal wavelets. It is quite localized and decline from their central peak. The Meyer wavelet’s discrete format approximation is dmey wavelet.¹²

METHODOLOGY

De-noising algorithms which uses the wavelet transform comprise of three steps.

- To find the wavelet transform of the noisy signal.
- To change the noisy wavelet coefficients as per prerequisite regulation.

- To find the inverse transform using the changed coefficients.

In this study, Wavelet Transform (WT) is used for denoising the medical image corrupted by Gaussian noise. Here, medical image is taken as an input. The Gaussian noise is generated randomly and added to the medical image. And then, WT is applied. The decomposition level used in this study is 2 and different wavelets are used.

Steps used in the study

- Step 1 - Take a medical image
- Step 2 - Add Gaussian noise
- Step 3- Take the wavelet transforms of noisy image using different wavelets.

original image



Fig. 5 (a) Original Ultrasound Ovary Image

Noise variance = 0.05

Noisy Image	Wavelets				
	1. Harr	2. db4	3. sym4	4. bior6.8	5. dmey
noisy image 	Denoised Img 	Denoised Img 	Denoised Img 	Denoised Img 	Denoised Img

Fig. 5 (b)

Noise variance = 0.09

Noisy Image	Wavelets				
	1. Harr	2. db4	3. sym4	4. bior6.8	5. dmey
noisy image 	Denoised Img 	Denoised Img 	Denoised Img 	Denoised Img 	Denoised Img

Fig. 5 (c)

Fig. 5(b-c). Ultrasound Denoised Images by various wavelets

Step 4 - Change the noisy wavelet coefficients as per the requirement.

Step 5 - Take inverse wavelet transform of changed wavelet coefficients.

Step 6 - Find the PSNR, SNR and MSE of denoised output image. Also find ET.

Experimental Setup

Programming language used for coding is MATLAB 2020B. A standard MRI of brain, chest X-ray, CT image, Ultrasound image of ovary and retina image of right are used for study of denoising medical images as shown in Fig. 1(a), Fig. 2(a), Fig. 3(a), Fig. 4(a) and Fig. 5(a) respectively. Gaussian

noise having noise variance of 0.05 and 0.09 was introduced in the medical image. Wavelet transform was performed using 5 different wavelets like Harr, db4, sym4, bior6.8 and dmey. The above process is repeated 3 times for different noise variance and each time SNR, PSNR, RMSE and execution time (ET) were calculated.

RESULTS AND DISCUSSION

Table 1-3 displays the denoising parameters in terms of SNR, PSNR, RMSE and execution time for different wavelets. Fig. 2

original image

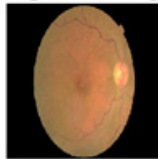


Fig. 6(a) Retina Image of right eye

Noise variance = 0.05

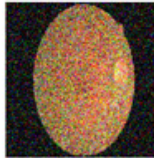

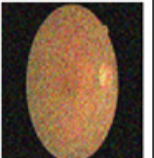
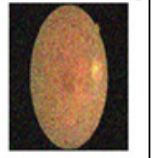
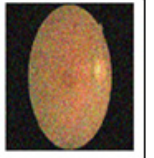
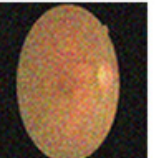
Noisy Image	Wavelets				
	1. Harr	2. db4	3. sym4	4. bior6.8	5. dmey
noisy image 	Denoised Img 	Denoised Img 	Denoised Img 	Denoised Img 	Denoised Img 

Fig. 6(b)

Noise variance = 0.09

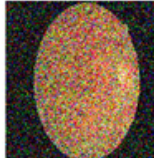
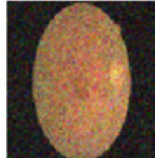
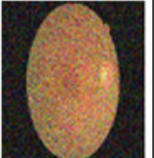
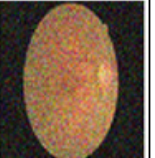
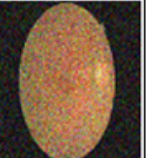
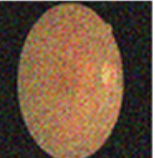
Noisy Image	Wavelets				
	1. Harr	2. db4	3. sym4	4. bior6.8	5. dmey
noisy image 	Denoised Img 	Denoised Img 	Denoised Img 	Denoised Img 	Denoised Img 

Fig. 6(c)

Fig. 6(b-c). Retina Image Denoised by various wavelets

(a-d), Fig. 3 (a-d), Fig. 4 (a-d), Fig. 5 (a-d) and Fig. 6 (a-d) show the pictures of various types of original medical images, noisy and denoised image. Multi-modal medical images were denoised using different wavelets. In original multi-modal medical images Gaussian noise having 0.05 and 0.09 were introduced. And these images were denoised using

DWT with the help of 5 different types of wavelets. It can be seen from the denoised images obtained that the denoising parameters vary with different wavelets. For all noise variance dmey wavelet reduces more noise in medical image and gives better SNR, PSNR and RMSE. Similar results are obtained for other medical images MRI, X-ray,

Table 1(A-B). Impact of wavelets on noise parameters for MRI

Table 1(A). Noise – Gaussian, Noise variance = 0.05

Sr No.	Denoising parameters	Noisy Img parameters	Wavelets				
			1. Harr	2. db4	3. sym4	4. bior6.8	5. dmey
1	SNR	4.0437	7.9324	8.3532	8.4618	8.3467	8.6454
2	PSNR	15.0559	20.8025	21.1497	21.2693	21.0797	21.5160
3	RMSE	2029.9907	540.5470	499.0087	485.4548	507.1149	458.6440
4	ET	-	0.41398	0.28123	0.28123	0.28114	0.35632

Table 1(B). Noise - Gaussian, Noise variance = 0.09

Sr No.	Denoising parameters	Noisy Img parameters	Wavelets				
			1. Harr	2. db4	3. sym4	4. bior6.8	5. dmey
1	SNR	2.9341	6.4510	6.7814	6.8106	6.6776	6.9646
2	PSNR	12.7990	18.8227	19.0594	19.1279	18.8505	19.3417
3	RMSE	3413.3158	852.7227	807.4905	794.858	847.2929	756.6783
4	ET	-	0.77915	0.61345	0.59719	0.39631	0.41275

Table 2(A-B). Impact of wavelets on noise parameters for X-ray Images

Table 2(A). Noise - Gaussian, Noise variance = 0.05

Sr No.	Parameters	Noisy Img	Wavelets used for Denoising				
			1. Harr	2. db4	3. sym4	4. bior6.8	5. dmey
1	SNR	9.3853	19.1203	19.4645	19.4237	19.0484	19.5193
2	PSNR	13.7855	24.0645	24.4097	24.3636	23.9780	24.4668
3	RMSE	2719.7337	255.0541	235.5623	238.0782	260.1845	232.4894
4	ET	-	0.36336	0.4189	0.38673	0.41855	0.5505

Table 2(B). Noise - Gaussian, Noise variance = 0.09

Sr No.	Parameters	Noisy Img	Wavelets used for Denoising				
			1. Harr	2. db4	3. sym4	4. bior6.8	5. dmey
1	SNR	7.5677	16.9453	17.1583	17.1124	16.7643	17.1952
2	PSNR	11.7936	22.0125	22.2171	22.1761	21.8061	22.2647
3	RMSE	4302.4759	409.1013	390.2775	393.9790	429.0143	386.0212
4	ET	-	0.2636	0.31234	0.28042	0.32132	0.35584

Table 3(A-B). Impact of wavelets on noise parameters for CT Images

Table 3(A). Noise - Gaussian, Noise variance = 0.05

Sr No.	Parameters	Noisy Img	Wavelets used for Denoising				
			1. Harr	2. db4	3. sym4	4. bior6.8	5. dmey
1	SNR	9.1962	15.3155	16.0597	16.0784	16.2372	16.4595
2	PSNR	14.4796	21.1647	21.8846	21.9034	22.0605	22.3032
3	RMSE	2318.0148	497.2923	421.3312	419.5055	404.6026	382.6146
4	ET	-	0.62455	0.48287	0.53743	0.54176	0.96824

Table 3(B). Noise - Gaussian, Noise variance = 0.09

Sr No.	Parameters	Noisy Img	Wavelets used for Denoising				
			1. Harr	2. db4	3. sym4	4. bior6.8	5. dmey
1	SNR	7.3470	13.4375	13.8991	19.8693	14.0043	14.2971
2	PSNR	12.4464	19.3944	19.8476	19.8693	19.9352	20.2705
3	RMSE	3702.0695	747.5473	673.4656	670.1102	660.0218	610.9850
4	ET	-	0.43939	0.84431	0.40388	0.80107	0.97444

Table 4(A-B). Impact of wavelets on noise parameters for ultrasound Images

Table 4(A). Noise - Gaussian, Noise variance = 0.05

Sr No.	Parameters	Noisy Img	Wavelets used for Denoising				
			1. Harr	2. db4	3. sym4	4. bior6.8	5. dmey
1	SNR	4.5936	10.7668	11.2147	11.2146	10.7295	11.3593
2	PSNR	14.6838	22.5272	22.9535	22.9629	22.4096	23.1100
3	RMSE	2211.5762	363.3780	329.4073	328.6930	373.3553	317.7486
4	ET	-	0.47157	0.4751	0.56423	0.61324	0.80855

Table 4(B). Noise - Gaussian, Noise variance = 0.09

Sr No.	Parameters	Noisy Img	Wavelets used for Denoising				
			1. Harr	2. db4	3. sym4	4. bior6.8	5. dmey
1	SNR	3.4562	8.9853	9.2309	9.2561	8.8204	9.3208
2	PSNR	12.5075	20.3061	20.5466	20.5669	20.0463	20.6487
3	RMSE	3650.3444	605.9993	573.3477	570.6795	643.3523	560.0318
4	ET	-	0.5723	1.4247	0.57397	0.48878	0.76511

Table 5(A-B). Impact of wavelets on noise parameters for Retina Image**Table 5(A).** Noise - Gaussian, Noise variance = 0.05

Sr No.	Parameters	Noisy Img	Wavelets used for Denoising				
			1. Harr	2. db4	3. sym4	4. bior6.8	5. dmey
1	SNR	6.6038	12.7675	12.8269	13.0170	12.5426	13.2154
2	PSNR	14.6275	21.7209	21.7690	21.9560	21.4382	22.1894
3	RMSE	2240.4053	437.5167	432.6892	414.4578	466.9343	392.7747
4	ET	-	0.17465	0.22211	0.17608	0.20391	0.2504

Table 5(B). Noise - Gaussian, Noise variance = 0.09

Sr No.	Parameters	Noisy Img	Wavelets used for Denoising				
			1. Harr	2. db4	3. sym4	4. bior6.8	5. dmey
1	SNR	4.9742	10.7800	10.8362	10.9240	10.3908	11.0438
2	PSNR	12.4453	19.6432	19.6716	19.7959	19.2130	19.9504
3	RMSE	3702.9407	705.9231	701.3186	681.5370	779.4333	657.7212
4	ET	-	0.26397	0.27853	0.3458	0.31123	0.63357

CT, Ultrasound and retinal image. But, execution time of dmey wavelet is more as compare to other wavelets.

CONCLUSION

Many diseases are diagnosed by using medical imaging methods like CT scan, X-ray, ultrasound, MRI, etc. But, noise degrades the quality of images and it becomes difficult to diagnose the disease. Here, the experimental results clearly shows for Gaussian noise the dmey wavelet gives better SNR, PSNR and RMSE than Harr, db4, sym4 and bior6.8 wavelets. But, its execution time is higher than other wavelets. Similar result is obtained for other five medical images i.e. MRI, X-ray, CT, Ultrasound and retina image studied for experimentation. So, it can be concluded that for Gaussian noise removal dmey wavelet gives better results for multi-modal medical images.

In this proposed solution, only Gaussian noise is used for analysis. Similarly, impact of other types of noises like speckle, Salt and Pepper noise etc. which also affect multi-modal medical images should be also studied and compared on the basis of SNR PSNR, RMSE and execution time.

Conflict of interest

There are no conflict of interest.

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