

Optimum Global Thresholding Based Variable Block Size DCT Coding For Efficient Image Compression

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ABSTRACT

Image compression is dissimilar than compressing other unprocessed binary data. Obviously, general purpose compression techniques deliberated for raw binary data can be used to compress images, but the result is not the finest one, because of the variations in statistical property of image. Statistical properties of image must be completely exploit by encoders to obtained efficient image compression. The most popular transform coder was developed by the Joint Photographic Experts Group (JPEG), which utilizes fixed block size discrete cosine transform (DCT) to obtain image data decorrelation. With fixed block size DCT, JPEG has no independent control over quality and bit rate simultaneously. Human visual system is less sensitive to quantization noise in high-activity areas of an image than in low-activity areas. That means the threshold of visibility of quantization noise is higher in areas with details than in flat areas. this is the key, why not exploit human visual system's weakness to achieve much higher compression with good visual quality by hiding as much quantization noise in busy areas as possible. It is indeed possible by using DCT with varying block sizes and quantizing busy areas heavily and flat areas lightly. To apply variable block size DCT transform on an image, quad tree decomposition technique is commonly used. In quad tree decomposition technique a square block is divides in smaller blocks, if the difference between maximum and minimum pixel values exceeds a threshold. This threshold selection plays a very crucial role, because the independent selection of threshold value without considering the statistical properties of the input image, may lead to even worst compression characteristics. To address this difficulty, this paper proposes a novel optimum global thresholding based variable block size DCT coding for efficient image compression. The proposed method calculates the required threshold value for blocks decomposition using optimum global thresholding technique, which exploits the edge characteristics of the image. Comparison has been made with baseline fixed block size DCT coder by using mean square error (MSE), peak signal to noise ratio (PSNR) and compression ratio (CR) as criterions. It is shown that the variable block size DCT transform coding system using proposed optimum global thresholding technique has better MSE and highly improved PSNR and CR performance for all test images.

Key words: Variable block size DCT, quad-tree decomposition, optimum global threshold, MSE, CR, PSNR.

INTRODUCTION

In the past few decades, transform coding has been used widely in compressing image and video data^{1,2,3}. A typical transform coding system⁴ divides an input image into fixed size square blocks. The frequently used block sizes are 8x8 or 16x16. At the

transmitter, every block is transformed by an orthogonal transform. The DCT is regularly preferred because of its high potential of energy compaction and its fast computational algorithm. The transform coefficients are then quantized and coded for transmission. At the receiver, the process is inverted to obtain the decoded image. In most existing

transform coding systems like JPEG, the block size used to divide an input image is fixed. This approach, however, has not taken into account, that image statistics may be inhomogeneous and may vary from area to area in an image. Some areas of an image can have only soft changes and contain no large contrast edge. In these areas, larger compression can be obtained by using a larger block size. However, some areas contain high activities and contrast edges, smaller block size transform, must be used to gain better visual quality¹. Therefore, to truly adapt to the internal statistics of an image in different areas, a transform coding system ought to vary the block size to give up a better tradeoff between the bit rate and the quality of decoded image. Generally speaking, if a segment of an image contains high activities, the segment should be partitioned into smaller areas. This process continues until the divided segments have homogeneous statistics or only smooth changes.

In⁵, an adaptive transform coding system using variable block size was proposed. The system uses a mean-difference based criterion to determine whether a block contains high contrast edges or not. If a block contains high contrast edges, the block is divided into four smaller blocks and the process repeats with the divided blocks until the four blocks contain no further high contrast edges or the smallest block size is reached.

In⁷, a classified vector quantization (CVQ) of an image, based on quad trees and a classification technique in the discrete cosine transform (DCT) domain was proposed. They have obtained decoded images of good visual quality for encoding rates of 0.3 and 0.7 bpp.

In⁸, this work describes a progressive image transmission (PIT) scheme using a variable block size coding technique in conjunction with a variety of transform domain quantization schemes. The developed scheme utilized a region growing technique to partition the images so that regions of different sizes can be addressed using a small amount of side information. Simulation results shown that the reconstructed images preserve fine and pleasant qualities based on both subjective and mean square error criteria.

In⁹ an edge-oriented progressive image

coding scheme using a hierarchical edge extraction was presented. This scheme is based on the two-component model, that is, edges and smooth component. It is shown through the simulations that the proposed scheme results in performance improvements over MPEG-2 I-picture coding in terms of both the subjective quality and signal-to-noise ratio.

In¹⁰ variable image segmentation and mixed transform were used to capitalize on the narrow band and broadband signal components for image compression. A gain of around 1.7-5 dB in PSNR over conventional JPEG was reported by authors.

This paper proposes a novel optimum global thresholding based variable block size DCT coding for efficient image compression. The proposed method calculates the required threshold value for blocks decomposition using optimum global thresholding technique, which exploits the edge characteristics of the image. Comparison has been made with baseline fixed block size DCT coder by using Mean Square Error (MSE) and compression ratio (CR) as criterions. It is shown that the variable block size DCT transform coding system using proposed optimum global thresholding technique has better MSE, PSNR and CR performance for all test images. In next section, the fixed and variable block size DCT transform coding system is briefly described. The proposed optimum global thresholding based variable block size DCT coding, is then defined in next section. To compare the performance of the system using two different criteria, Results and discussion section gives the simulation result obtained for three images coded at the quality factor 5. Finally, last section is the conclusion.

Fixed and variable block size dct image coding **Fixed Block Size DCT Image Coding**

In case of fixed block size DCT coding, to apply the DCT, the input image is divided into 8'8 blocks of pixels. If the width or height of the input image is not divisible by 8, the encoder must make it divisible. The 8'8 blocks are processed from left-to-right and from top-to-bottom¹¹.

The purpose of the DCT is to transform the value of pixels to the spatial frequencies. These spatial frequencies are extremely linked to the level of detail present in an image. High spatial frequencies provide

high levels of detail, while lower frequencies give lower levels of detail. The mathematical definition of DCT is^{11, 19, 20}:

Forward DCT

$$F(u, v) = \frac{1}{4} C(u)C(v) \sum_{x=0}^7 \sum_{y=0}^7 f(x, y) \cos \left[\frac{\pi(2x+1)u}{16} \right] \cos \left[\frac{\pi(2y+1)v}{16} \right] \quad (2.1)$$

For $u=0, \dots, 7$ and $v=0, \dots, 7$.

$$\text{Where } C(k) = \begin{cases} 1/\sqrt{2} & \text{for } k=0 \\ 1 & \text{otherwise} \end{cases}$$

Inverse DCT

$$f(x, y) = \frac{1}{4} \sum_{u=0}^7 \sum_{v=0}^7 C(u)C(v) F(u, v) \cos \left[\frac{\pi(2x+1)u}{16} \right] \cos \left[\frac{\pi(2y+1)v}{16} \right] \quad (2.2)$$

For $x=0, \dots, 7$, and $y=0, \dots, 7$.

The $F(u, v)$ is called DCT coefficient, while DCT basis is given as:

$$\omega_{x,y}(u, v) = \frac{C(u)C(v)}{4} \cos \left[\frac{\pi(2x+1)u}{16} \right] \cos \left[\frac{\pi(2y+1)v}{16} \right] \quad \dots(2.3)$$

Then we can rewrite the inverse DCT to:

$$f(x, y) = \sum_{u=0}^7 \sum_{v=0}^7 F(u, v) \omega_{x,y}(u, v) \quad \dots(2.4)$$

For $x=0, \dots, 7$, and $y=0, \dots, 7$.

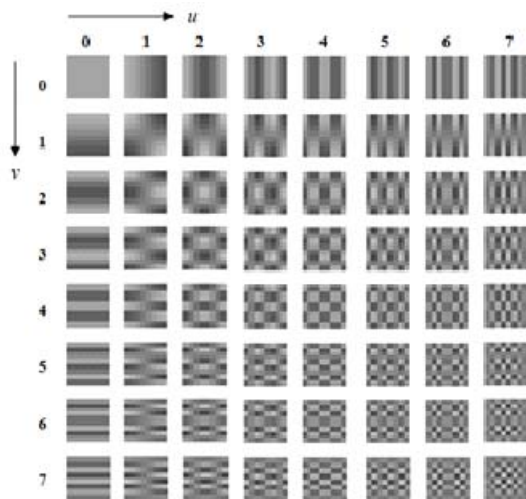


Fig. 1: The 8'8 DCT basis $\omega_{x,y}(u, v)$

The transformed 8'8 block now consists of 64 DCT coefficients. The very first coefficient $F(0,0)$ is the DC component and the next 63 coefficients are AC component. The DC component $F(0,0)$ is basically the sum of the 64 pixels in the input 8'8 pixel block multiplied by the scaling factor $(1/4)C(0)C(0)=1/8$ as shown in equation 3 for $F(u, v)$.

The next step in the compression process is to quantize the transformed coefficients. Each one of the 64 DCT coefficients are uniformly quantized [11, 14, and 15]. The 64 quantization step-size parameters for uniform quantization of the 64 DCT coefficients form an 8'8 quantization matrix. Every element in the quantization matrix is an integer values between ranges 1 to 255. Each DCT coefficient $F(u, v)$ is divided by the corresponding quantizer step-size parameter $Q(u, v)$ in the quantization matrix and then rounded to the nearest integer as:

$$F_q(u, v) = \text{Round} \left(\frac{F(u, v)}{Q(u, v)} \right) \quad \dots(2.5)$$

The JPEG standard does not define any fixed quantization matrix. It is the choice of the user to select a quantization matrix. There are two structures of the quantization matrices provided in JPEG standard. These two quantization matrices are shown below:

16	11	10	16	24	40	51	61
12	12	14	19	26	58	60	55
14	13	16	24	40	57	69	56
14	17	22	29	51	87	80	62
18	22	37	56	68	109	103	77
24	35	55	64	81	104	113	92
49	64	78	87	103	121	120	101
72	92	95	98	112	100	103	99

17	18	24	47	99	99	99	99
18	21	26	66	99	99	99	99
24	26	56	99	99	99	99	99
47	66	99	99	99	99	99	99
99	99	99	99	99	99	99	99
99	99	99	99	99	99	99	99
99	99	99	99	99	99	99	99
99	99	99	99	99	99	99	99

Fig. 2: Luminance and Chrominance Quantization matrix

The quantization process has the key role in the JPEG compression. It is the procedure which removes the high frequencies present in the input image. Quantization is used because of the fact that the eye is much more sensitive to lower spatial frequencies than to higher frequencies of the transformed image. This is performed by dividing values at high indexes in the vector (the amplitudes of higher frequencies) with larger values than the values by which are divided the amplitudes of lower frequencies. The larger values in the quantization table is the larger error introduced by this lossy process, and the smaller visual quality¹².

Variable Block Size DCT Image Coding

Previous subsection, briefly dealt with transform coder with fixed DCT block size. With fixed block size DCT, the coder doesn't have independent control over quality and bit rate simultaneously. Human visual system is less sensitive to quantization noise in high-activity areas of an image than in low-activity areas. That means that the threshold of visibility of quantization noise is higher in areas with details than in flat areas. In this case the basic idea is to exploit our own visual system's weakness to achieve much higher compression with good visual quality by hiding as much quantization noise in busy areas as possible. It is indeed possible if we use DCT with varying block sizes and quantize busy areas heavily and flat areas lightly¹².

One possible method is to start with an $N \times N$ block with $N = 2m$, m being a positive integer. Typically, N is 16 or 32. Using the block variance as the metric, we can decompose the $N \times N$ block into four $(N/2) \times (N/2)$ blocks. Each block, in turn, may be subdivided further into four blocks depending on the variance metric. This process of subdividing may be continued until left with 2×2 sub blocks. This is the familiar quad tree decomposition.

Once the quad tree decomposition is done, DCT can be applied to each sub block and the DCT coefficients quantized using suitable quantization matrices. DCTs of smaller blocks may be quantized rather heavily and of bigger sub blocks lightly to achieve higher compression without sacrificing the quality. This is feasible because smaller blocks were obtained on the basis of the variance; smaller blocks have higher variances than bigger blocks. Therefore, quantization

noise will be less visible in those smaller sub blocks due to the human visual response¹².

Optimum Global Thresholding

Thresholding may be considered as a statistical-decision theory problem whose objective is to minimize the average error incurred in assigning pixels to two or more groups (also called classes).

The approach of optimum global thresholding is also known as Otsu's method. The method is optimum in the sense that it maximizes the between-class variance, a famous measure used in statistical discriminant analysis. The basic idea is that thresholded classes must be distinct with respect to the intensity values of their pixels and, conversely, that a threshold giving the best separation between classes in terms of their intensity values would be the best (optimum) threshold. Besides to its optimality, Otsu's method has the important property that it is, based entirely on computations performed on the histogram of an image, an easily obtainable 1-D array¹³.

Let denote the L distinct intensity levels in a digital image of size pixels, and let , denote the number of pixels with intensity i . The algorithm of the optimum global thresholding technique is given as¹³:

i. Determine the normalized histogram of the input image. Denote the components of histogram by p_i , for $i=0,1,2,\dots,L-1$.

ii. Determine the cumulative sums, $P_1(k)$, for $k=0,1,2,\dots,L-1$. given as

$$P_1(k) = \sum_{i=0}^k p_i \quad \dots(2.6)$$

iii. Determine the cumulative means $m(k)$, for $k=0,1,2,\dots,L-1$. given as

$$m(k) = \sum_{i=0}^k i p_i \quad \dots(2.7)$$

iv. Determine the global intensity mean m_σ as

$$m_\sigma = \sum_{i=0}^{L-1} i p_i \quad \dots (2.8)$$

v. Calculate the between- class variance, $\sigma_B^2(k)$ for $k=0,1,2,\dots,L-1$. given as

$$\sigma_s^2(k) = \frac{[m_\sigma P_1(k) - m(k)]^2}{P_1(k)[1 - P_1(k)]} \dots(2.9)$$

vi. Obtain the Otsu threshold, k^* as the value of for which $\sigma_s^2(k)$ is maximum. If the unique maximum is not available, obtain by averaging the values of corresponding to the various maxima detected.

vii. Obtain the separability measure η^* , evaluating following at $k=k$.

$$\eta(k) = \frac{\sigma_s^2(k)}{\sigma_\sigma^2} \dots(2.10)$$

where σ_σ^2 is the global variance.

Proposed optimum global thresholding based variable block size dct coding

This section briefly presents the proposed optimum global thresholding based variable block size DCT image coding system. The proposed system starts with the use of quad tree decomposition, to divide an input image into sub blocks of size between 2x2 and 16x16 pixels. The block decomposition used here is based on the homogeneity of pixels in a block. The quad tree decomposition divides a square block if the difference between maximum and minimum pixel values exceeds a threshold (say T). Otherwise, the block is not divided further. The range of the threshold for block decomposition is . This threshold selection plays a very crucial role, because the independent selection of threshold value without considering the statistical properties of the input image, may lead to even worst compression characteristics.

To address this difficulty, this work utilized optimum global thresholding, to calculate the optimum threshold T, which leads accurate decision about the blocks division in sub blocks. Once the image is quad decomposed, the sub blocks are DCT transformed, DCT coefficients are quantized and dequantized, and finally inverse DCT transformed.

For the 8x8 blocks, the proposed work uses the default JPEG luma quantization matrix and quantizer scale. For the 2x2 and 4x4 blocks, heavy quantization is used, and for the 16x16 blocks, light quantization is used be used. The DC coefficient of the 2x2 and 4x4 blocks is quantized using a step size of 8. The AC coefficients of 2x2 blocks are quantized using a step size of 34, while those of 4x4 blocks are quantized using 24 as the step size. The

DC coefficient of the 16x16 blocks is quantized with a step size of 4 and all AC coefficients of the 16x16 blocks is quantized with a step size of 16. The complete process involved in the proposed coder is shown in figure (3), with the help of block diagram representation.

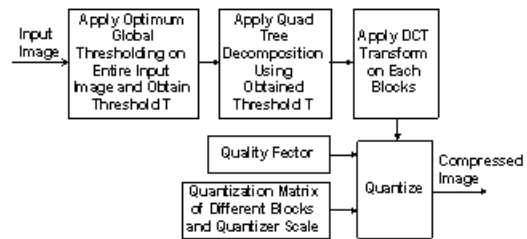


Fig. 3: Block diagram representation of the proposed image coder

RESULTS AND DISCUSSIONS

The proposed work has been successfully implemented and simulated on MATLAB 2012b. Simulation is carried out to compare the fixed block size DCT coding (FBSDCT) and proposed optimum globally threshold based variable block size DCT coding (OGTBVBSDCT) systems. The input image has resolution of 8 bits per pixel and the image size is. The largest and smallest block sizes allowed are 16x16 and 2x2 respectively. The decision threshold T is obtained optimum global threshold technique and the quality scale is fixed to 5. 50 images, has been used for comparative analysis, few of them (test images) have been shown in from Fig. 4.1 to Fig. 4.5. For the comparative analysis the three well known parameters compression ratio (CR), mean square error (MSE) and peak signal to noise ratio have been used. The resultant images obtained after using FBSDCT, and OGTBVBSDCT have been shown in Table-1

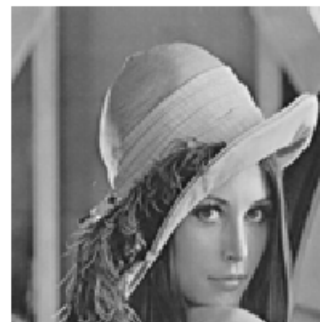


Fig. 4(a): Lena Image

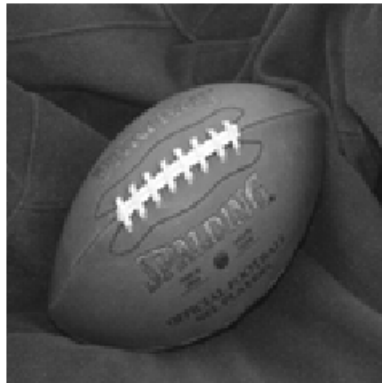


Fig. 4(2): Football Image



Fig. 4(3): Barbara Image

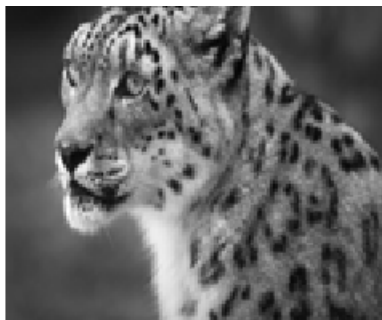


Fig. 4(4): Tiger Image

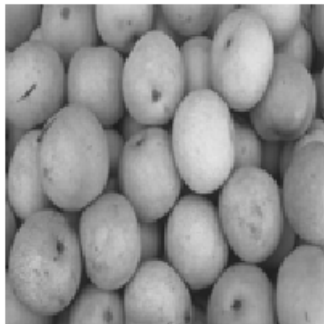


Fig. 4(5): Pears Image

Table 1: The resultant images obtained after using FBSDCT and proposed OGTBVBSDCT techniques for quality value Q = 5

Lena Image	Decompressed Lena Image Using FBSDCT
Quad Tree Decomposition for proposed coder for Lena Image	Decompressed Lena Image Using OGTBVBSDCT
Football Image	Decompressed Football Image Using FBSDCT
Quad Tree Decomposition for proposed coder for Football Image	Decompressed Football Image Using OGTBVBSDCT
Barbara Image	Decompressed Barbara Image Using FBSDCT
Quad Tree Decomposition for proposed coder for Barbara Image	Decompressed Barbara Image Using OGTBVBSDCT
Barbara Image	Decompressed Barbara Image Using FBSDCT
Quad Tree Decomposition for proposed coder for Barbara Image	Decompressed Barbara Image Using OGTBVBSDCT

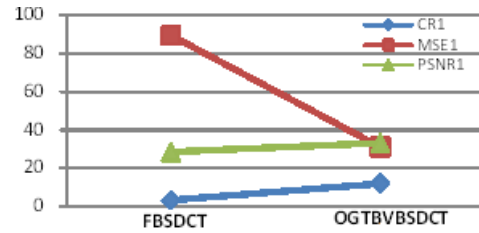
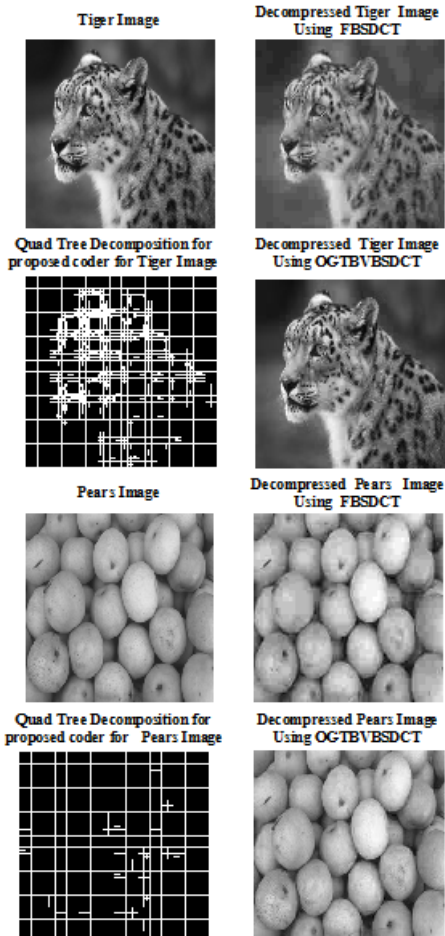


Fig. 6: Plot of CR, MSE and PSNR for Leena Image

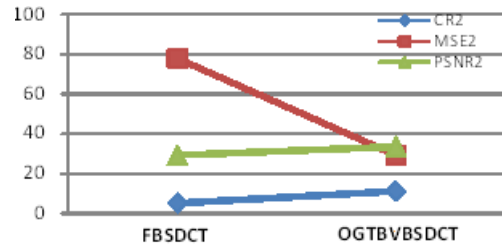


Fig. 7: Plot of CR, MSE and PSNR for Football Image

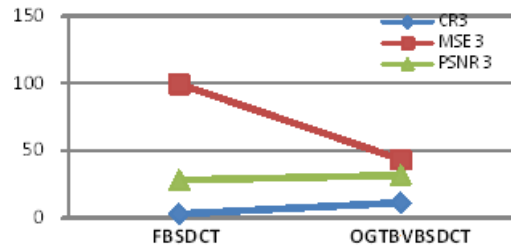


Fig. 8: Plot of CR, MSE and PSNR for Barbara Image

Table 2: Values of CR, MSE and PSNR for all five test images

Images	Parameters	FBS-DCT	Proposed OGTVBVS-DCT	Threshold Value (T) Obtained Using Optimum Global Thresholding
Lena Image	CR1	3.3572	12.0515	0.4627
	MSE1	89.386	31.1507	
	PSNR1	28.618	33.1961	
Football Image	CR2	5.1449	11.2085	0.3882
	MSE2	77.937	29.192	
	PSNR2	29.213	33.4782	
Barbara Image	CR3	3.0181	11.274	0.4392
	MSE3	99.356	43.2813	
	PSNR3	28.158	31.7678	
Tiger Image	CR4	2.9805	9.7495	0.4353
	MSE4	112.19	48.0093	
	PSNR4	27.631	31.3176	
Pears Image	CR5	3.7758	13.9706	0.502
	MSE5	57.947	20.5174	
	PSNR5	30.500	35.0096	

Now the values of compression ratio (CR), mean square error (MSE) and peak signal to noise ratio (PSNR), obtained after image compression using FBSDCT and proposed OGTVBSDCT image coding systems for all the input images shown from Fig. 4.1 to Fig. 4.5, have been tabulated in Table-2 and are plotted in Fig. 4.6 to Fig. 4.10.

It is clearly observable from Table-II and Fig. 4.6 to Fig. 4.10, that the developed technique OGTVBSDCT provides much higher compression

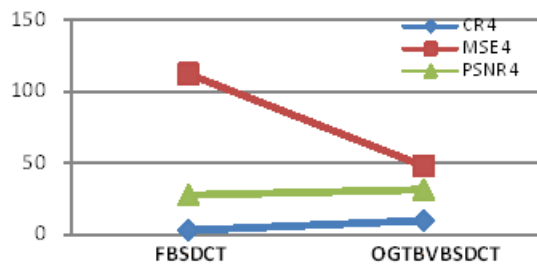


Fig. 9: Plot of CR, MSE and PSNR for Tiger Image

ratio as compared to available FBSDCT system. Close observation of obtained results reveals that, due to utilization of variable block size DCT with efficiently calculated threshold "T" using optimum global thresholding technique, enables the developed image coder to greatly reduce the blocking artifacts during decompression, clearly reflected by the higher value of CR, PSNR and lower value of MSE in the plots, which was not the case of available FBSDCT system. Hence the developed is able to obtain efficient image compression than the available FBSDCT system.

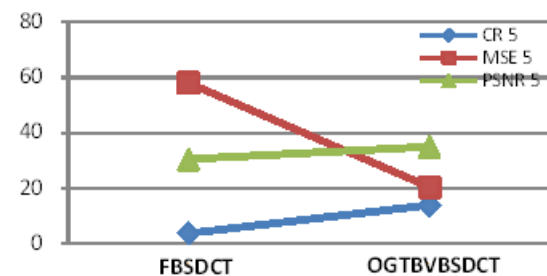


Fig. 10: Plot of CR, MSE and PSNR for Pears Image

CONCLUSIONS

In this paper a novel optimum global thresholding based variable block size DCT coding for Efficient Image compression has been successfully implemented using MATLAB. The proposed method calculates the required threshold value for blocks decomposition efficiently using optimum global thresholding technique, which exploits the edge characteristics of the image. This process is very crucial, because lower values of threshold "T" leads to higher number of block decomposition and higher values of "T" leads to smaller or non decomposition of blocks.

Most often higher value of "T" tends to convert variable block size DCT to fixed block size DCT. Therefore appropriate value of "T" must be required to maintain efficiency of block decomposition. A complete comparative analysis of the proposed system, based on 50 standard image database(out

of which five image comparisons are included in this paper), with baseline fixed block size DCT coder by using mean square error (MSE), peak signal to noise ratio (PSNR) and compression ratio (CR) as criterions, is also presented in the result section.

It is shown that the variable block size DCT transform coding system using proposed optimum global thresholding technique has better MSE and highly improved PSNR and CR performance for all test images. The proposed coding system OGTVBSDCT has achieved four times more compression ratio (CR) and restricted the MSE to fifty percent as that obtained for the available FBSDCT system. Hence the developed optimum global thresholding based variable block size DCT coding system (OGTVBSDCT) provides efficient image compression than the available fixed block size DCT coding system (FBSDCT). The proposed method can be further improved using various optimization techniques to get more refined threshold value for blocks decomposition.

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